Knox Grammar School

2016

Trial Higher School Certificate Examination

Name: ____________________

Teacher: ____________________

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
  Black pen is preferred
- Board-approved calculators may be used
- The official BOSTES Reference Sheet is provided
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Teachers:
Mr Bradford
Ms Yun
Mr Vuletich
Mr Mulray

Section I: Pages 1-4

10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section
- Use the Multiple Choice Answer Sheet

Section II: Pages 5-11

60 marks
- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section
- Answer each question in a separate writing booklet

Total marks – 70

Write your Name, your Board of Studies Student Number and your Teacher's Name on the front cover of each writing booklet

This paper MUST NOT be removed from the examination room

Number of Students in Course: 58

Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 The netball coach at a school has decided that the junior netball team will consist of:

4 students from Year Ten;

3 students from Year Nine.

After tryouts, there were 8 eligible Year Ten students and 9 eligible Year Nine students left to select from. In how many ways can the team be selected?

(A) ${}^{8}\mathbf{C}_4 + {}^{9}\mathbf{C}_3$

(B) ${}^{8}\mathbf{P}_4 + {}^{9}\mathbf{P}_3$

(C) ${}^{17}\mathbf{C}_7$

(D) ${}^{8}\mathbf{C}_4 \times {}^{9}\mathbf{C}_3$

2 What is the number of asymptotes on the graph of $y = \dfrac{1}{x^2 - 1}$?

(A) 1

(B) 2

(C) 3

(D) 4

3 It is known that when the polynomial $P(x) = x^3 + 3x^2 + 2x$ is divided by $(x - a)$ the remainder is zero.

What values could a take?

(A) $-3, \; -2, \; 0$

(B) $-2, \; -1, \; 0$

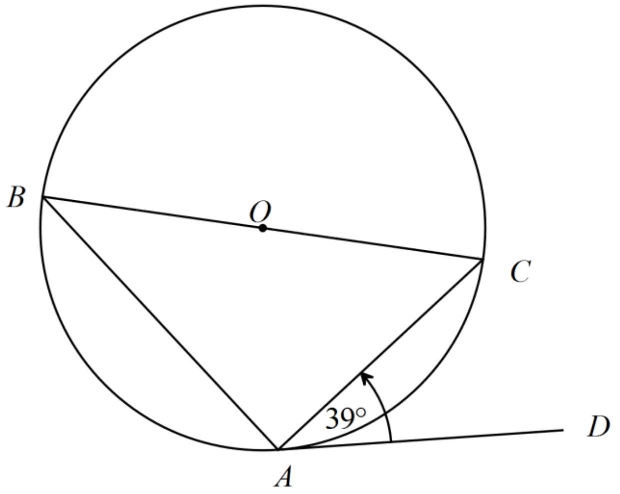
(C) $-2, \; 1, \; 3$

(D) $0, \; 1, \; 2$

4 In the circle, centre O, BC is a diameter.

AD is a tangent to the circle with A being the point of contact.

$\angle DAC = 39°$.



What is the size of $\angle BCA$?

(A) 39°

(B) 51°

(C) 78°

(D) 89°

5 Evaluate $\lim_{x \to 0} \dfrac{\sin 4x}{3x}$.

(A) 0

(B) $\dfrac{3}{4}$

(C) $\dfrac{4}{3}$

(D) ∞

6 What is the domain of the function $y = \sin^{-1}(x + 5)$?

(A) $-6 \leq x \leq -4$

(B) $-5 \leq x \leq 5$

(C) $-\frac{\pi}{5} \leq x \leq \frac{\pi}{5}$

(D) $\frac{\pi}{6} \leq x \leq \frac{\pi}{4}$

7 Find $\int \frac{1}{9 + 25x^2}\,dx$.

(A) $\frac{1}{15}\tan^{-1}\frac{5x}{3} + C$

(B) $\frac{1}{25}\tan^{-1}\frac{5x}{3} + C$

(C) $\frac{1}{25}\tan^{-1}\frac{3x}{5} + C$

(D) $\frac{1}{15}\tan^{-1}\frac{3x}{5} + C$

8 A curve has parametric equations $x = t - 3$ and $y = t^2 + 2$.

What is the Cartesian equation of this curve?

(A) $y = x^2 - x - 1$

(B) $y = x^2 + x - 1$

(C) $y = x^2 - 6x + 11$

(D) $y = x^2 + 6x + 11$

9 A particle moves in simple harmonic motion such that $v^2 + 9x^2 = k$.

What is the period of the particle's motion?

(A) $\dfrac{2\pi}{k}$

(B) 3π

(C) $\dfrac{3k}{2\pi}$

(D) $\dfrac{2\pi}{3}$

10 Which inequality has the same solution as $|x-1|+|x+2|=3$?

(A) $\dfrac{1}{x-1}-\dfrac{1}{x+2}\geq 0$

(B) $x^2+x-2\geq 0$

(C) $\dfrac{3}{1-x}\leq 2$

(D) $|2x+1|\leq 3$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Find $\int \sin^2 x \, dx$. **1**

(b) Calculate the size of the acute angle between the lines $2x + y = 5$ and $3x - y = 1$. **2**

(c) Solve the inequality $\dfrac{2}{1+3x} \leq 1$. **3**

(d) Find the coordinates of the point P that divides the interval joining the points $A(-1,4)$ and $B(5,-5)$ externally in the ratio 1:2. **2**

(e) Express $4\cos x - 3\sin x$ in the form $R\cos(x+\alpha)$, where $0 \leq \alpha \leq 90^\circ$. **2**

(f) Find the coefficient of x^{14} in the binomial expansion of $\left(2x + \dfrac{1}{3x}\right)^{18}$. **2**

(g) Use the substitution $u = x - 3$ to evaluate $\int_3^4 x\sqrt{x-3}\, dx$. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) When the polynomial $P(x) = x^4 - 8x^3 - 7x^2 + 3$ is divided by $x^2 + x$, **2**

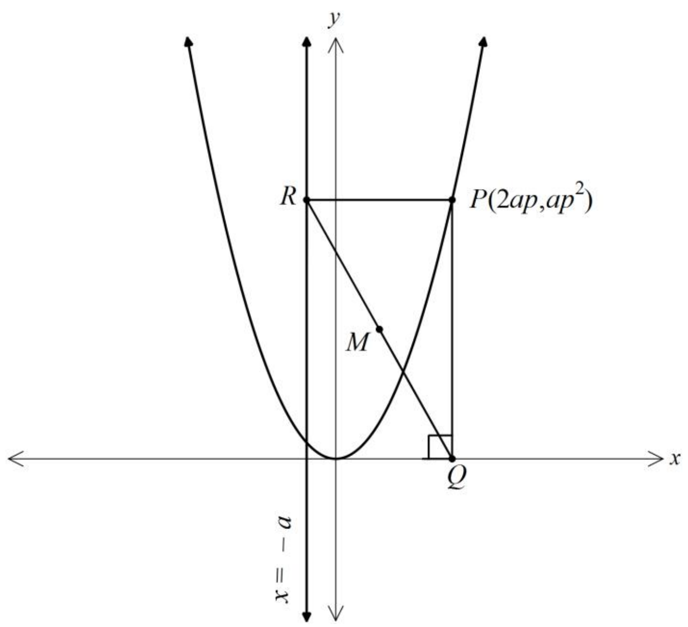
the remainder is $ax + 3$, where a is constant. Find the value of a.

(b) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$.

The point Q is a point on the x-axis such that PQ is parallel to the y-axis.

The point R is a point on the line $x = -a$ such that RP is parallel to the x-axis.

M is the midpoint of interval RQ.



(i) Show that M has coordinates $\left(\dfrac{a(2p-1)}{2}, \dfrac{ap^2}{2}\right)$. **2**

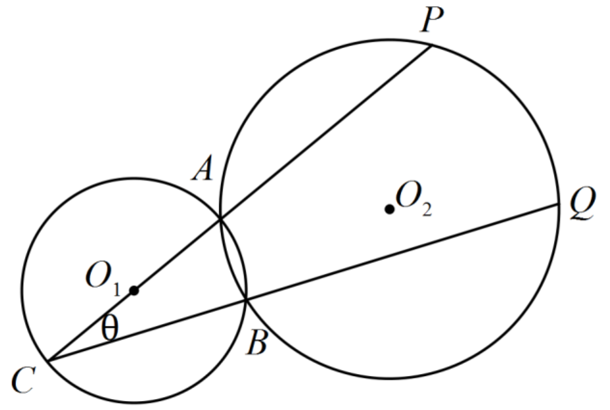
(ii) Show that the locus of the point M is a parabola with equation **2**

$$y = \frac{x^2}{2a} + \frac{x}{2} + \frac{a}{8}.$$

(iii) Find the equation of the axis of symmetry for the parabola **1**
which forms the locus of M.

Question 12 continues on the following page

(c) Two circles with centres O_1 and O_2 intersect at points A and B as shown in the diagram.



AC is a diameter of the circle with centre O_1 and it intersects the other circle at A and P. The chord CB produced intersects the second circle again at Q. Let $\angle ACB = \theta$.

Copy or trace the diagram into your writing booklet.

(i) Prove that AQ is a diameter of the circle with centre O_2. **2**

(ii) Show that $\angle ABO_1 = 90 - \theta$. **2**

(d) A group of 10 people, consisting of 6 girls and 4 boys, decided to go to the movies where they sit together in the same row.

(i) How many different arrangements of seating are possible where the 4 boys are all seated together? **2**

(ii) If they are seated in random order, what is the probability that at least one of the boys is separated from the other boys? **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The function $f(x) = \sec x$ is defined for $0 \leq x < \frac{\pi}{2}$.

(i) Show that $f(x)$ is monotonic increasing in this domain. **1**

(ii) Show that $f^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$. **1**

(iii) Hence find $\frac{d}{dx} f^{-1}(x)$. **1**

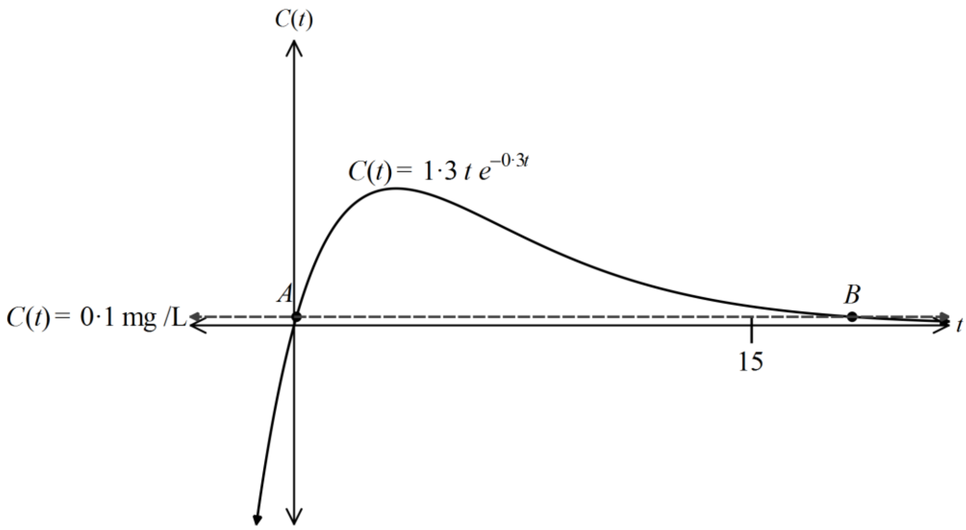
(b) A patient was administered with a drug.

The concentration of the drug in the patient's blood followed the rule:

$$C(t) = 1.3t\, e^{-0.3t}$$

where time, t, is measured in hours and $C(t)$ is measured in mg/L.

This rule is graphed below.



The doctor left instructions that the patient must not receive another dose of the medicine until the concentration of the drug had dropped to below 0.1 mg/L.

(i) Using $t = 15$ as a first approximation, use one application of Newton's method to find approximately when the concentration of the drug in the blood of the patient reaches 0.1 mg/L. **2**

(ii) Would it be appropriate to use your answer in (i) as the time when the drug would next be administered? Explain your answer **1**

Question 13 continues on the following page

(c) What are the roots of the equation $x^3 + 6x^2 - x - 30 = 0$ given one root is the sum of the other two roots? **3**

(d) A particle with velocity v m/s and displacement x m from the origin is moving in a straight line governed by the equation $v^2 = -2x^2 - 8x - 6$.

(i) Prove that the particle is moving in simple harmonic motion. **1**

(ii) Find the amplitude of the motion. **2**

(e) Use mathematical induction to prove that for all integers $n \geq 3$ **3**

$$\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)\ldots\left(1-\frac{2}{n}\right) = \frac{2}{n(n-1)}.$$

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Six students are using the same brand of pen in their examinations.

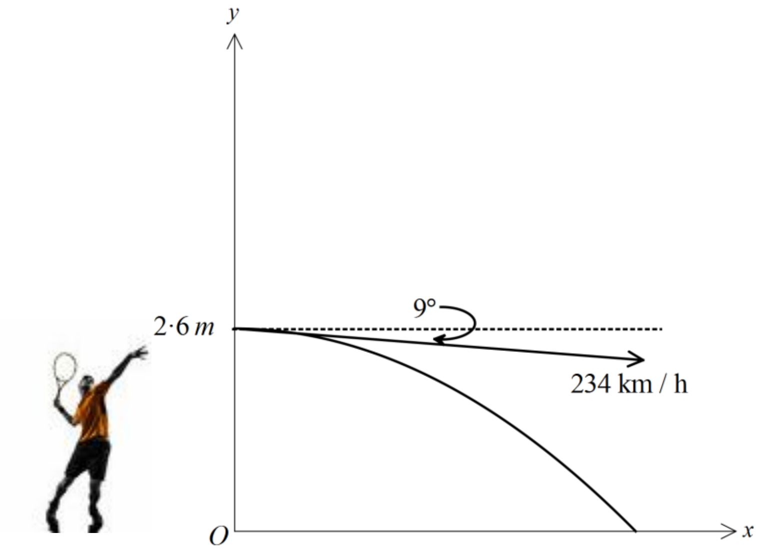
They all have four examinations to complete and only one pen each.

The probability that a single pen will last through four examinations is 0.65.

(i) What is the probability (correct to 4 decimal places) that exactly two of the pens will last through the four exams? **1**

(ii) What is the probability (correct to 4 decimal places) that at least two of the pens will last through all four examinations. **1**

(b) Sam served a tennis ball to his opponent. The racquet hit the ball when the ball was 2.6 metres above the ground. The initial speed of the ball is 234 km/h at an angle of 9° below the horizontal.



Let the origin be a point on the ground, directly below where the racquet hit the ball.

The only force acting on the particle is due to gravity where g is $10\ \text{m/s}^2$.

(i) Show that the motion of the ball (in metres) can be expressed by the equations **2**

$$x = 65t\cos 9° \quad \text{and} \quad y = 2.6 - 5t^2 - 65t\sin 9°.$$

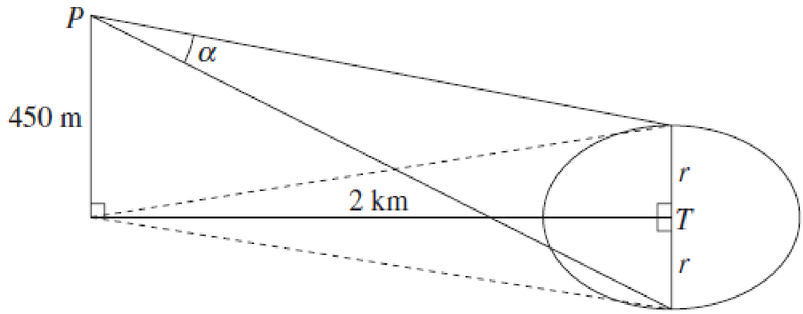
(ii) The net at the centre of the court is 11.9 metres from the origin. **2**

The net is 91cm tall. Show that the ball will not make it over the net.

(iii) What will be the speed of the ball when it hits the net? **1**

Question 14 continues on the following page

(c) An oil tanker at T is leaking oil which forms a circular oil slick. An observer is measuring the oil slick from a position P, 450 metres above sea level and 2 kilometres horizontally from the centre of the oil slick.



(i) At a certain time the observer measures the angle, α, subtended by the diameter of the oil slick, to be 0.1 radians. What is the radius, r, at this time? **2**

(ii) At this time, $\frac{d\alpha}{dt} = 0.02$ radians per hour. Find the rate at which the radius of the oil slick is growing. **2**

(d) (i) Use the binomial expansion for $(1+x)^n$ to show that

$$\left\{\sum_{r=0}^{n} {}^nC_r \frac{x^{r+1}}{r+1}\right\} + \frac{1}{n+1} = \frac{(1+x)^{n+1}}{n+1} \quad .$$ **2**

(ii) Hence find $\sum_{r=0}^{n} (-1)^r \frac{{}^nC_r}{(r+1)(r+2)}$ giving your answer as a simple expression in terms of n. **2**

End of paper

2016 Year 12 Mathematics Extension 1 Task 5 SOLUTIONS

Suggested Solution (s)	Comments
Question 1 ${}^8C_4 \times {}^9C_3 \quad \therefore D$	
Question 2 Vertical $x = \pm 1$ Horizontal as $x \to \infty$ $y \to 0$ $\therefore y = 0$ $\therefore$ 3 asymptotes $\quad \therefore C$	
Question 3 $P(x) = x(x^2+3x+2)$ $= x(x+1)(x+2)$ $P(a) = 0$ $\therefore a(a+1)(a+2) = 0$ $a = 0, -1, -2$ $\therefore B$	
Question 4 $\angle ABC = 39^\circ$ ($\angle$ in alternate segment) $\angle BAC = 90^\circ$ ($\angle$ in semi circle) $\therefore \angle BCA = 180^\circ - 90^\circ - 39^\circ$ $= 51^\circ$ $\therefore B$	
Question 5 $\lim_{x \to 0} \frac{\sin 4x}{3x}$ $= \frac{4}{3} \lim_{x \to 0} \frac{\sin 4x}{4x}$ $= \frac{4}{3} \times 1 = \frac{4}{3} \quad \therefore C$	
Question 6 $P: \ -1 \leq x+5 \leq 1$ $\therefore -6 \leq x \leq -4 \quad \therefore A$	
Question 7 $\frac{1}{25}\int \frac{1}{\frac{9}{25}+x^2}\,dx$ $= \frac{1}{25}\int \frac{1}{(\frac{3}{5})^2+x^2}\,dx$ $= \frac{1}{25} \times \frac{1}{\frac{3}{5}} \tan^{-1}\left(\frac{x}{\frac{3}{5}}\right) + c$ $= \frac{1}{15}\tan^{-1}\left(\frac{5x}{3}\right) + c$ $\therefore A$	
Question 8 $y = (x+3)^2 + 2$ $y = x^2 + 6x + 11 \quad \therefore D$	
Question 9 $v^2 = k - 9x^2$ $= 9\left(\frac{k}{9} - x^2\right)$ $= n^2(a^2 - x^2)$ where $n^2 = 9$ $\quad a^2 = \frac{k}{9}$ $n = \pm 3$ $T = \frac{2\pi}{n} = \frac{2\pi}{3} \quad \therefore D$	
Question 10 Noting $x = 1$ is a solution of $\lvert x-1\rvert + \lvert x+2\rvert = 3$ then alternatives (A) and (C) are not possible as in both of these $x \neq 1$ In (B) $\quad (x+2)(x-1) \geq 0$ $x \leq -2$ or $x \geq 1$  , but $x = 0$ is also a solution of $\lvert x-1\rvert + \lvert x+2\rvert = 3$ $\therefore$ (B) is not possible This leaves only (D) $\therefore D$	

2016 Year 12 Mathematics Extension 1 Task 5 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 11 (a) $\int \sin^2 x\,dx$ $= \int \frac{1}{2} - \frac{1}{2}\cos 2x\,dx$ $= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$ ✓	– no penalty for no + C	(d) $A(-1,4)$ $B(5,-5)$ (x_1, y_1; x_2, y_2) $k:l = -1:2$ (external) $P\left(\frac{kx_2+lx_1}{k+l}, \frac{ky_2+lx_1}{k+l}\right)$ $= P\left(\frac{-1\times 5+2(-1)}{-1+2}, \frac{-1\times(-5)+2(4)}{-1+2}\right)$ ✓ $= P(-7, 13)$ ✓	
(b) $m_1 = -2$, $m_2 = 3$ $\tan\theta = \left\|\frac{m_1 - m_2}{1+m_1 m_2}\right\|$ $= \left\|\frac{-2-3}{1+(-2)(3)}\right\|$ $= \left\|\frac{-5}{-5}\right\|$ $= 1$ ✓ $\therefore \theta = 45^\circ$ ✓		(e) $R = \sqrt{4^2+3^2}$ $R = 5$ ✓ $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ ✓ $\doteqdot 36^\circ 52'$ (f) $T_{r+1} = {}^{18}C_r (2x)^{18-r}(3x)^{-r}$ $= {}^{18}C_r\, 2^{18-r}\cdot 3^{-r}\, x^{18-2r}$ ✓ For x^{14} let $r = 2$ $\therefore$ Coeff ${}^{18}C_2\, 2^{16}\cdot 3^{-2}$ ✓ (1114112)	
(c) $\frac{2}{1+3x} \le 1 \quad (x \ne -\frac{1}{3})$ $\times(1+3x)^2$: $2(1+3x) \le (1+3x)^2$ $(1+3x)^2 - 2(1+3x) \ge 0$ $(1+3x)(1+3x-2) \ge 0$ $(1+3x)(3x-1) \ge 0$ $x \le -\frac{1}{3}$ or $x \ge \frac{1}{3}$  but $x \ne -\frac{1}{3}$ $\therefore x < -\frac{1}{3}$ or $x \ge \frac{1}{3}$	✓ ✓ ✓	(g) $\int_3^4 x\sqrt{x-3}\,dx$ $du = dx$ $x=3, u=0$ $x=4, u=1$ $= \int_0^1 (u+3)u^{\frac{1}{2}}\,du$ $= \int_0^1 u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\,du$ $= \left[\frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}}\right]_0^1$ $= \frac{2}{5} + 2 = 2\frac{2}{5}$	✓ limits ✓ integral in terms of u ✓ value

2016 Year 12 Mathematics Extension 1 Task 5 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 12			

Suggested Solution (s)

Question 12

(a) $P(x) = x(x+1)Q(x) + ax + 3$

$P(-1) = -a + 3$

also $P(-1) = (-1)^4 - 8(-1)^3 - 7(-1)^2 + 3$

$= 5$

Comments: ✓

$\therefore -a + 3 = 5$

$a = -2$

Comments: ✓

(b)(i) $R(-a, ap^2)$ $Q(2ap, 0)$

Comments: ✓ coords of R and Q

$\therefore M\left(\dfrac{-a+2ap}{2}, \dfrac{ap^2+0}{2}\right)$

Comments: ✓ use in midpoint formula

$\therefore M\left(\dfrac{a(2p-1)}{2}, \dfrac{ap^2}{2}\right)$

(ii) $x = \dfrac{a(2p-1)}{2}$

$\therefore \dfrac{2x}{a} = 2p - 1$

$p = \dfrac{1}{2} + \dfrac{x}{a}$ ✓

sub in $y = \dfrac{ap^2}{2}$

$= \dfrac{a}{2}\left(\dfrac{x}{a} + \dfrac{1}{2}\right)^2$

$= \dfrac{a}{2}\left(\dfrac{x^2}{a^2} + \dfrac{x}{a} + \dfrac{1}{4}\right)$

$= \dfrac{x^2}{2a} + \dfrac{x}{2} + \dfrac{a}{2}$

Comments: ✓ progress to result

iii) $x = \dfrac{-B}{2A}$ $\quad B = \dfrac{1}{2}$ $\quad A = \dfrac{1}{2a}$

$x = \dfrac{-\frac{1}{2}}{2 \times \frac{1}{2a}} = -\dfrac{1}{2} \times \dfrac{a}{1}$

$\therefore x = -\dfrac{a}{2}$ ✓

(c)(i) Construct AB

$\therefore \angle ABC = 90°$ ($\angle$ in semi circle)

Join PQ

$\therefore APQ = 90°$ ($\angle$ in cyclic quad BQPA equal to remote opp. $\angle$)

Comments: 1

$\therefore \angle APQ = 90°$ is angle in semi circle

$\therefore AQ$ must be a diameter.

Comments: 1

or $\angle ABQ = 90°$ (Adjacent supplementary angle) and AQ must be diameter

ii) Since

$\angle CAB = 90 - \theta$ (Angle sum of $\triangle ABC$) ✓

and $O_1A = O_1B$ (Equal Radii)

Then $AB_1O = 90 - \theta$

(Base $\angle$'s of isosceles triangles are equal) ✓

(d) i) Taking 4 boys as 1 element

placing 7 elements (7! ways) ✓

However amongst themselves 4 boys can be arrange 4! ways

$\therefore 7! \times 4! = 120960$ ✓

(ii) If at least 1 boy separated, we are considering the probability the four boys will not be seated together.

$\therefore P(\text{at least 1 boy separated})$

$= 1 - P(\text{4 boys together})$ ✓

$= 1 - \dfrac{7!4!}{10!} = \dfrac{29}{30}$ ✓

2016 Year 12 Mathematics Extension 1 Task 5 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 13

(a) i) $f'(x) = \sec x \tan x \geq 0$ as $\sec x > 0$ and $\tan x \geq 0$ for $0 \leq x < \frac{\pi}{2}$

$\therefore$ Monotonic Increasing ✓

ii) $f: \; y = \sec x \quad 0 \leq x < \frac{\pi}{2}$

$f^{-1}: \; x = \sec y \quad 0 \leq y < \frac{\pi}{2}$

$x = \frac{1}{\cos y}$

$\cos y = \frac{1}{x}$

$\therefore y = \cos^{-1}\left(\frac{1}{x}\right)$

✓ progress to result

iii) $\frac{d}{dx}\cos^{-1}\left(\frac{1}{x}\right) = \frac{-1}{\sqrt{1-\frac{1}{x^2}}} \times \frac{-1}{x^2}$

$= \frac{1}{x\sqrt{x^2-1}}$ ✓

(b) let

i) $f(t) = 1.3te^{-0.3t} - 0.1$

$f'(t) = 1.3t \times -0.3e^{-0.3t} + e^{-0.3t} \times 1.3$

$= -0.39te^{-0.3t} + 1.3e^{-0.3t}$ ✓

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}, \quad x_1 = 15$

$= 15 - \frac{f(15)}{f'(15)}$

$= 17.307$ hours ✓

ii) $C(17.307) = 0.125$ mg/L > 0.1 mg/L

$\therefore$ Not appropriate

c) let $\alpha = \beta + \gamma$

Sum: $\beta + \gamma + \beta + \gamma = -6$

$\beta + \gamma = -3$ ✓

product: $(\beta+\gamma)\beta\gamma = 30$

$-3\beta\gamma = 30$

$\beta\gamma = -10$

$\gamma = \frac{-10}{\beta}$

$\beta - \frac{10}{\beta} = -3$

$\beta^2 + 3\beta - 10 = 0$

$(\beta+5)(\beta-2) = 0$

$\beta = -5, 2$

$\gamma = 2, -5$

$\alpha = -3$ ✓ ✓

$\therefore$ Roots are $-5, -3, -2$

(d) i) $\frac{1}{2}v^2 = -x^2 - 4x - 3$

$\ddot{x} = -2x - 4$

$= -2(x+2)$ ✓

which is SHM about $x = -2$.

ii) End points when $v = 0$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -1, -3$ ✓

Centre $x = -2$

$\therefore$ Amplitude is 1m. ✓

2016 Year 12 Mathematics Extension 1 Task 5 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 13 (e) When $n=3$ $L.H.S = 1-\frac{2}{3} = \frac{1}{3}$ $R.H.S = \frac{2}{3(3-1)} = \frac{1}{3}$ $\therefore$ Statement true for $n=3$ Assume statement true for $n=k$, some fixed positive integer i.e. $(1-\frac{2}{3})(1-\frac{2}{4})(1-\frac{2}{5})\dots(1-\frac{2}{k}) = \frac{2}{k(k-1)}$ When $n=k+1$ $L.H.S = (1-\frac{2}{3})(1-\frac{2}{4})(1-\frac{2}{5})\dots(1-\frac{2}{k})(1-\frac{2}{k+1})$ $= \frac{2}{k(k-1)}\cdot(1-\frac{2}{k+1})$ by assumption $= \frac{2}{k(k-1)}\cdot\left(\frac{k+1-2}{k+1}\right)$ $= \frac{2(k-1)}{k(k-1)(k+1)}$ $= \frac{2}{(k+1)k}$ $= \frac{2}{n(n-1)}$ when $n=k+1$ $= R.H.S$	✓ ✓ progress to result. ✓		

2016 Year 12 Mathematics Extension 1 Task 5 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 14

(a) (i) let $p = 0.65$ (pen lasts)

$q = 0.35$ (pen doesn't last)

$n = 6$

$$P(X=2) = {}^6C_2 (0.35)^4 (0.65)^2 \checkmark$$

$$= 0.0951$$

No penalty for rounding.

ii) $P(X \geq 2) = 1 - \left[P(X=0) + P(X=1)\right]$

$$= 1 - \left[{}^6C_0 (0.35)^6 + {}^6C_1 (0.35)^5 (0.65)^1\right] \checkmark$$

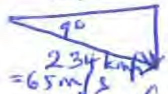
$$= 0.9777 \text{ (4 d.p)}$$

(b) $\ddot{x} = 0$ $\quad \ddot{y} = -10$

(i) Initially $t = 0$

$\dot{x}_0 = 65\cos(-9)$

$\dot{y}_0 = 65\sin(-9)$



$\dot{x} = \int \ddot{x}\, dt$

$= C$

$= 65\cos(-9)°$

$= 65\cos 9°$

$\dot{y} = \int -10\, dt$

$= -10t + C$

$t = 0,\ \dot{y} = 65\sin(-9)$

$= -65\sin 9°$

$\therefore \dot{y} = -10t - 65\sin 9°$

$x = \int \dot{x}\, dt$

$= \int 65\cos 9°\, dt$

$= 65t\cos 9° + C$

at $t = 0$, $x = 0$

$\therefore x = 65t\cos 9°$

$y = \int \dot{y}\, dt$

$= -5t^2 - 65t\sin 9° + C$

at $t = 0$, $y = 2.6$

$\therefore y = 2.6 - 5t^2 - 65t\sin 9°$

2 marks for correct working leading to correct expressions for x and y.
1 mark for an incomplete derivation or incorrect derivation due to an error in reasoning, calculation or algebra in the working.

ii) at $x = 11.9$ m

$11.9 = 65t\cos 9°$

$$t = \frac{11.9}{65\cos 9°} \checkmark$$

$$\therefore y = 2.6 - 5\left(\frac{11.9}{65\cos 9°}\right)^2 - 65 \times \left(\frac{11.9}{65\cos 9°}\right)\sin 9°$$

$= 0.54$ m (2 d.p) $\checkmark$

< 91 cm

$\therefore$ Ball will not clear net

iii) when $t = \dfrac{11.9}{65\cos 9°}$

$\dot{x}_t = 65\cos 9° = 64.199\ldots$ m/s

$$\dot{y}_t = -10\left(\frac{11.9}{65 \times \cos 9°}\right) - 65\sin 9° = -12.0\ldots$$

$$v_t = \sqrt{\dot{x}_t^2 + \dot{y}_t^2}$$

$$= \sqrt{(64.199\ldots)^2 + (-12.025\ldots)^2}$$

$= 65.32$ m/s $\checkmark$

(c) (i) Join PT

$\therefore PT^2 = 2^2 + 0.45^2$

$PT = 2.05$ km.

$$\therefore \tan\left(\frac{\alpha}{2}\right) = \frac{r}{2.05}$$

$\alpha = 0.1$ radians.

$$\therefore r = 2.05\tan\left(\frac{0.1}{2}\right)$$

$= 103$ m (to nearest metre)

2016 Year 12 Mathematics Extension 1 Task 5 SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 14 continued

ii) $r = 2.05\tan\left(\frac{\alpha}{2}\right)$

$\therefore \frac{dr}{d\alpha} = \frac{2.05}{2}\sec^2\left(\frac{\alpha}{2}\right)$ ✓

$\frac{dr}{dt} = \frac{dr}{d\alpha} \times \frac{d\alpha}{dt}$

when $\alpha = 0.1$

$\frac{dr}{dt} = \frac{2.05}{2}\sec^2\left(\frac{0.1}{2}\right) \times 0.02$

$= 0.02055\ldots$ km/h ✓ or equivalent

$= 20.6$ m/h

(d) (i)

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$

Integrating.

$\frac{(1+x)^{n+1}}{n+1} = {}^nC_0x + {}^nC_1\frac{x^2}{2} + {}^nC_2\frac{x^3}{3} + \ldots + {}^nC_n\frac{x^{n+1}}{n+1} + K$

at $x=0$, $K = \frac{1}{n+1}$

$\therefore {}^nC_0x + {}^nC_1\frac{x^2}{2} + {}^nC_2\frac{x^3}{3} + \ldots + {}^nC_n\frac{x^{n+1}}{n+1} + \frac{1}{n+1} = \frac{(1+x)^{n+1}}{n+1}$

as required.

✓ - integration

✓ - find K.

ii) Integrating again:

${}^nC_0\frac{x^2}{1\times 2} + {}^nC_1\frac{x^3}{2\times 3} + {}^nC_2\frac{x^4}{3\times 4} + \ldots$

${}^nC_n\frac{x^{n+2}}{(n+1)(n+2)} + \frac{1}{n+1}x + K_2$

$= \frac{(1+x)^{n+2}}{(n+1)(n+2)} \quad \ldots (*)$

at $x=0$ $K_2 = \frac{1}{(n+1)(n+2)}$ ✓

Now,

$\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{(r+1)(r+2)}$

$= \frac{{}^nC_0}{1\times 2} - \frac{{}^nC_1}{2\times 3} + \frac{{}^nC_2}{3\times 4} - \ldots + \frac{(-1)^n\,{}^nC_n}{(n+1)(n+2)}$

sub $x=-1$ in $(*)$

we have

$\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{(r+1)(r+2)}$

$= \frac{1}{n+1} - \frac{1}{(n+1)(n+2)}$

$= \frac{n+2-1}{(n+1)(n+2)}$

$= \frac{1}{n+2}$ ✓